Polynomial expression

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A polynomial expression S(x) in one variable x is an algebraic expression in x term as

$$S(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots \ldots \ldots \ldots + ax + a_0$$

Where $a_n, a_{n-1}, \ldots, a, a_0$ are constant and real numbers and a_n is not equal to zero

Some important points to remember

1) $a_n, a_{n-1}, a_{n-2}, \ldots.. a_1, a_0$ are called the coefficients for $x^n, x^{n-1}, \ldots.. x^1, x^0$

2) n is called the degree of the polynomial

3) when $a_n, a_{n-1}, a_{n-2}, \ldots.. a_1, a_0$ all are zero, it is called zero polynomial

4) A constant polynomial is the polynomial with zero degree, it is a constant value polynomial

5) A polynomial of one item is called monomial, two items binomial and three items as trinomial

6) A polynomial of one degree is called linear polynomial, two degree as quadratic polynomial and degree three as cubic polynomial

Zero's or roots of the polynomial

It is a solution to the polynomial equation S(x)=0 i.e. a number "a" is said to be a zero of a polynomial if S(a) = 0.
If we draw the graph of S(x) =0, the values where the curve cuts the X-axis are called Zeros of the polynomial

a) Linear polynomial has only one root

b) A zero polynomial has all the real number as roots

c) A constant polynomial has no zeros

Remainder Theorem's

If p(x) is an polynomial of degree greater than or equal to 1 and p(x) is divided by the expression (x-a),then the reminder will be p(a)

Factor's Theorem's

If x-a is a factor of polynomial p(x) then p(a)=0 or if p(a) =0,x-a is the factor the polynomial p(x)

Geometric Meaning of the Zero's of the polynomial

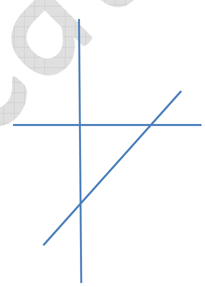
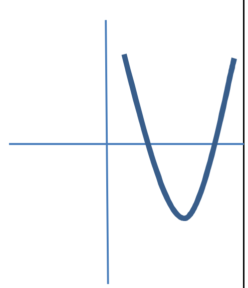
Lets us assume

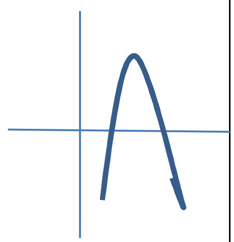
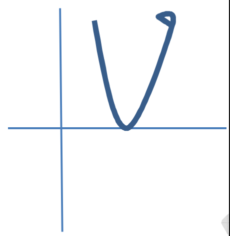
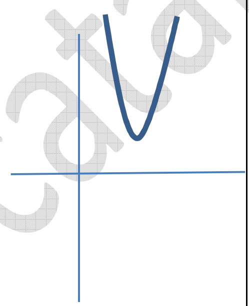
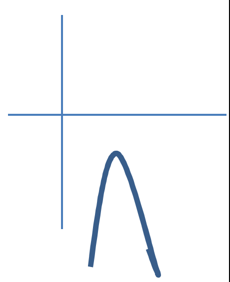
y= p(x) where p(x) is the polynomial of any form.

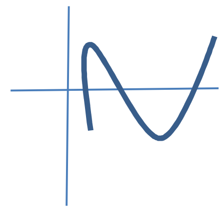
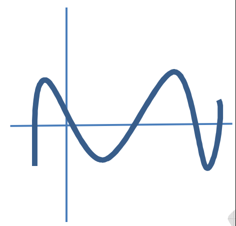
Now we can plot the equation y=p(x) on the Cartesian plane by taking various values of x and y obtained by putting the values. The plot or graph obtained can be of any shapes

The zero's of the polynomial are the points where the graph meet x axis in the Cartesian plane. If the graph does not meet x axis ,then the polynomial does not have any zero's.

Let us take some useful polynomial and shapes obtained on the Cartesian plane

S.no	y=p(x)	Graph obtained	Name of the graph	Name of the equation
1	$y=ax+b$ where a and b can be any values ($a\neq0$) Example $y=2x+3$		Straight line. It intersect the x-axis at (-b/a ,0) Example (-3/2,0)	Linear polynomial
2	$y=ax^2+bx+c$ where $b^2-4ac > 0$ and $a\neq0$ and $a> 0$ Example $y=x^2-7x+12$		Parabola It intersect the x-axis at two points Example (3,0) and (4,0)	Quadratic polynomial

3	$y=ax^2+bx+c$ where $b^2-4ac > 0$ and $a \neq 0$ and $a < 0$ Example $y=-x^2+2x+8$		Parabola It intersect the x-axis at two points Example (-2,0) and (4,0)	Quadratic polynomial
4	$y=ax^2+bx+c$ where $b^2-4ac = 0$ and $a \neq 0$ $a > 0$ Example $y=(x-2)^2$		Parabola It intersect the x-axis at one points	Quadratic polynomial
5	$y=ax^2+bx+c$ where $b^2-4ac < 0$ and $a \neq 0$ $a > 0$ Example $y=x^2-2x+6$		Parabola It does not intersect the x-axis It has no zero's	Quadratic polynomial
6	$y=ax^2+bx+c$ where $b^2-4ac < 0$ and $a \neq 0$ $a < 0$ Example $y=-x^2-2x-6$		Parabola It does not intersect the x-axis It has no zero's	Quadratic polynomial

7	y=ax^3 +bx^2+cx+d where a≠0	It can be of any shape 	It will cut the x-axis at the most 3 times	Cubic Polynomial
8	$a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots \dots \dots \dots + ax + a_0$ Where a_n ≠0	It can be of any shape 	It will cut the x-axis at the most n times	Polynomial of n degree

<u>*Relation between coefficient and zero's of the Polynomial:*</u>

S.no	Type of Polynomial	General form	Zero's	Relationship between Zero's and coefficients
1	Linear polynomial	ax+b , a≠0	1	$k = \frac{-constant\ term}{Coefficent\ of\ x}$
2	Quadratic	ax^2+bx+c, a≠0	2	$k_1 + k_2 = -\frac{Coefficent\ of\ x}{Coefficent\ of\ x^2}$ $k_1k_2 = \frac{Contant\ term}{Coefficent\ of\ x^2}$
3	Cubic	ax^3+bx^2+cx+d, a≠0	3	$k_1 + k_2 + k_3 = -\frac{Coefficent\ of\ x^2}{Coefficent\ of\ x^3}$ $k_1k_2k_3 = -\frac{Contant\ term}{Coefficent\ of\ x^{32}}$ $k_1k_2 + k_2k_3 + k_1k_3 = \frac{Coefficent\ of\ x}{Coefficent\ of\ x^2}$

Formation of polynomial when the zeros are given

Type of polynomial	Zero's	Polynomial Formed
Linear	k=a	(x-a)
Quadratic	k_1=a and k_2=b	(x-a)(x-b) Or x^2-(a+b)x +ab Or x^2-(Sum of the zero's)x +product of the zero's
Cubic	k_1=a ,k_2=b and k_3=c	(x-a)(x-b)(x-c)

Division algorithm for Polynomial

Let's p(x) and q(x) are any two polynomial with q(x) ≠0 ,then we can find polynomial s(x) and r(x) such that

P(x)=s(x) q(x) + r(x)

Where r(x) can be zero or degree of r(x) < degree of g(x)

Dividend =Quotient X Divisor + Remainder

Steps to divide a polynomial by another polynomial

1) Arrange the term in decreasing order in both the polynomial

2) Divide the highest degree term of the dividend by the highest degree term of the divisor to obtain the first term,

3) Similar steps are followed till we get the reminder whose degree is less than of divisor